THE UNIVERSITY OF WESTERN ONTARIO
FACULTY OF ENGINEERING SCIENCE
DEPARTMENT OF ELECTRICAL ENGINEERING

E.S. 438b COMPUTATIONAL ELECTROMAGNETICS

1996 Final Examination

Date:	**Saturday, April 20, 1996**	**Instructors: K. Adamiak**
Time:	**9:00-12:00 A.M.**	**J. LoVetri**
Location:	**Social Science Building, Room 3026**	

General Instructions:

1) This is an **open book** exam. Calculators **are** permitted, but **sharing** of calculators between students is **not** permitted.

2) Answer **all 5** questions in the examination booklet provided.

3) Each problem is worth **10** marks. Print clearly--illegible work will not be marked. Clearly indicate the steps taken in your answers and identify the final solution to each part of the question.

4) Make sure that your name, student number, and signature are written on the examination booklet.

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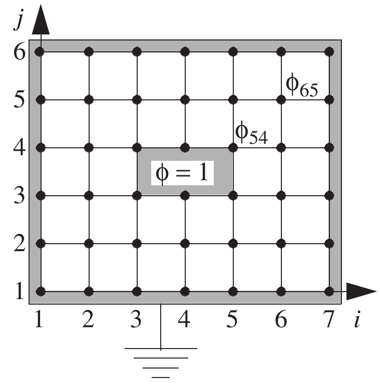
1) Consider the two dimensional electrostatic problem shown in the figure.

   a) Using the boundary conditions shown, determine the matrix equation (do not solve) which would result from applying a centered difference second order approximation to solve for the grid potentials, ϕ_{ij}, on the mesh. (Use the numbering scheme shown in the figure.)



   b) Discuss how you would use symmetry to reduce the size of the numerical problem.

   c) Using the grid potentials, ϕ_{ij}, write down a formula for the capacitance between the centre and outer conductors.

2) Consider the parallel-plate capacitor depicted in the figure with plate sizes much larger than the distance between them (*i.e.* a one-dimensional problem). The space between the plates is filled with a uniformly charged dielectric material having a relative permitivity of $\varepsilon_r = 2.0$ and a charge density of $\rho = 10^{-6}$ [C/m^3].

   a) Solve the problem analytically by integrating Poisson's equation.

   b) Discretize the domain into three equal finite elements.

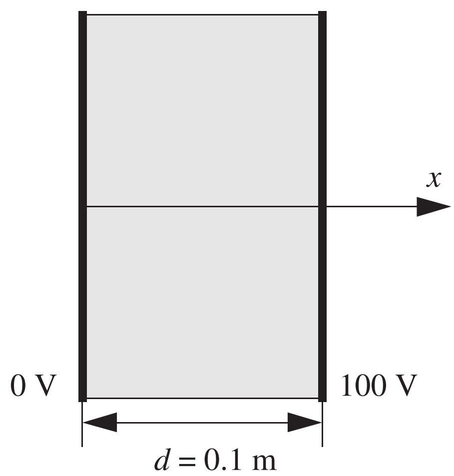
   c) Assuming the linear interpolation, derive the matrix [S] for each element.

   d) Assemble the set of algebraic equations for the nodal values of solution using the finite element method.

   e) Introduce boundary conditions.

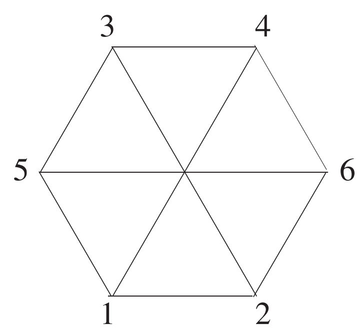
   f) Solve the set of algebraic equations.

   g) Compare analytical and numerical solutions at nodes.

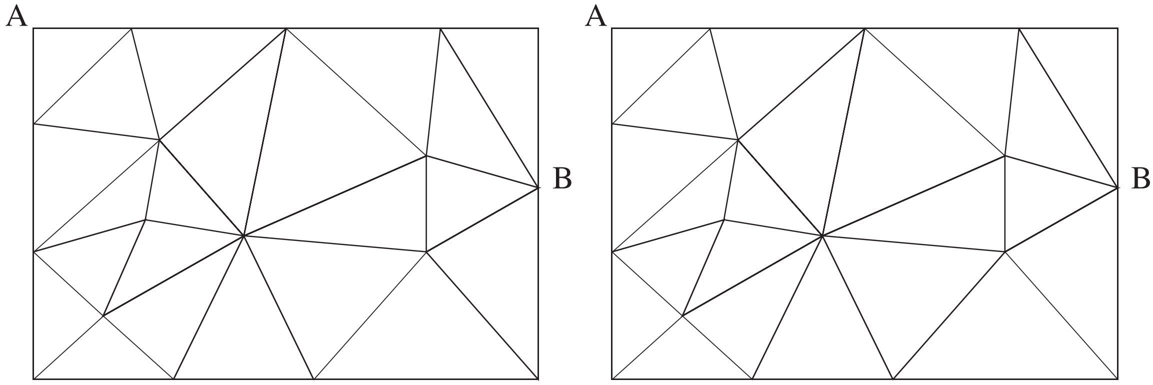


3) For the finite element mesh shown in in the figure, determine the total stiffness matrix [S] before and after introducing boundary conditions, assuming that the problem is governed by the Laplace equation in Cartesian coordinates and linear interpolation is used. All triangles are equilateral and the following Dirichlet boundary conditions are specified:

$$V_1 = 0,\ V_2 = 0,\ V_3 = 100 \text{ and } V_4 = 100.$$



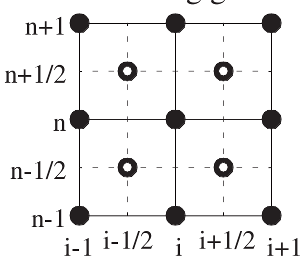
4) For the mesh given in the figure, number all nodes according to the Cuthill-McKee algorithm. Repeat the procedure twice treating nodes A and B as root nodes. In both cases determine the half-bandwith of the matrix.



5) The lossy transmission line equations can be written in matrix form in terms of the voltage, $V(x, t)$ [V], and the current, $I(x, t)$ [A], along the line as:

$$\frac{\partial}{\partial x}\begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix} + \begin{bmatrix} 0 & L \\ C & 0 \end{bmatrix}\frac{\partial}{\partial t}\begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix} + \begin{bmatrix} 0 & R \\ G & 0 \end{bmatrix}\begin{bmatrix} V(x,t) \\ I(x,t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

where L is the per unit length inductance [H/m], C is the per unit length capacitance [F/m], R is the per unit length resistance [Ω/m], and G is the per unit length conductance [S/m]. Consider the following grid functions and interlaced grid:



● $V_i^n = V(i\Delta x, n\Delta t)$

○ $I_{i+1/2}^{n+1/2} = I((i+1/2)\Delta x, (n+1/2)\Delta t)$

Using the following second order accurate centered difference approximations for the drivatives in the lossy transmission line equations:

$$\left.\frac{\partial V}{\partial x}\right|_{i+1/2}^{n} = \frac{V_{i+1}^n - V_i^n}{\Delta x} + O(\Delta x^2), \quad \left.\frac{\partial I}{\partial x}\right|_{i}^{n+1/2} = \frac{I_{i+1/2}^{n+1/2} - I_{i-1/2}^{n+1/2}}{\Delta x} + O(\Delta x^2),$$

$$\left.\frac{\partial V}{\partial t}\right|_{i}^{n+1/2} = \frac{V_i^{n+1} - V_i^n}{\Delta t} + O(\Delta t^2), \quad \left.\frac{\partial I}{\partial t}\right|_{i+1/2}^{n} = \frac{I_{i+1/2}^{n+1/2} - I_{i+1/2}^{n-1/2}}{\Delta t} + O(\Delta t^2),$$

(a) Determine a set of stable explicit update equations which approximate the above coupled partial differential equations. What is the stability limit for your scheme?

(b) Draw the computational molecule for the resulting scheme using solid dots for the voltage and hollow dots for the current.

(c) How does your scheme change at a boundary where the per unit length inductance, L, changes abruptly? Which component would you put at the boundary? (Draw this boundary on a sample grid.)